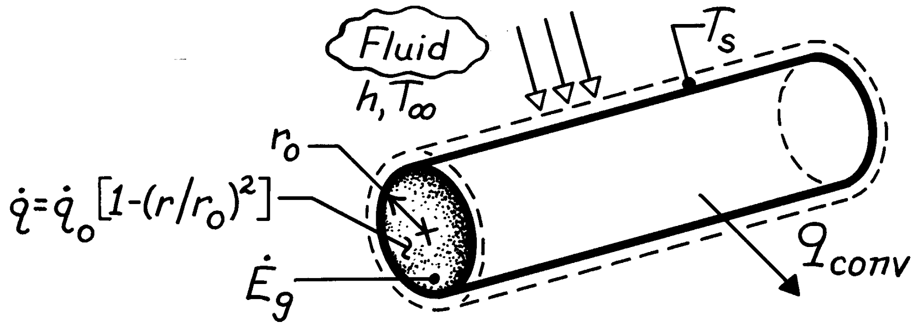
PROBLEM 1.39

KNOWN: Radial distribution of heat dissipation in a cylindrical container of radioactive wastes. Surface convection conditions.

FIND: Total energy generation rate and surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) Negligible temperature drop across thin container wall.

ANALYSIS: The rate of energy generation is

$$\dot{E}_g = \int \dot{q} dV = \dot{q}_o \int_0^{r_o} \left[1-\left(r/r_o\right)^2\right] 2\pi r L dr$$

$$\dot{E}_g = 2\pi L \dot{q}_o \left(r_o^2/2 - r_o^2/4\right)$$

or per unit length,

$$\dot{E}_g' = \frac{\pi \dot{q}_o r_o^2}{2}.$$ <

Performing an energy balance for a control surface about the container yields, at an instant,

$$\dot{E}_g' - \dot{E}_{out}' = 0$$

and substituting for the convection heat rate per unit length,

$$\frac{\pi \dot{q}_o r_o^2}{2} = h\left(2\pi r_o\right)\left(T_s - T_\infty\right)$$

$$T_s = T_\infty + \frac{\dot{q}_o r_o}{4h}.$$ <

COMMENTS: The temperature within the radioactive wastes increases with decreasing r from T_s at r_o to a maximum value at the centerline.